7. (a) Express $f(x)=\begin{cases}1 & \text{for } 0\le x\le \pi \\ 0 & \text{for } x>\pi\end{cases}$ as a Fourier sine integral and hence evaluate

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin x\lambda\, d\lambda .$$

(b) Find the Fourier transform of $f(x)=\dfrac{1}{\sqrt{|x|}}$.

8. (a) Solve the integral equation

$$\int_0^\infty f(\theta)\cos\alpha\theta\, d\theta=\begin{cases}1-\alpha & , \ 0\le\alpha\le 1 \\ 0 & , \ \alpha>1\end{cases}.$$ Hence evaluate $\displaystyle\int_0^\infty \frac{\sin^2 t}{t^2}dt$.

(b) State Parseval's identity for Fourier transforms and use it to prove that

$$\int_0^\infty \frac{t^2}{(t^2+4)(t^2+9)}dt=\frac{\pi}{10}.$$

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[05 BENG – 2101] ECE

II/IV B.Tech. DEGREE EXAMINATION. Oct-16

First Semester

Electronics and Communication Engineering

MATHEMATICS — IV

(Common for B.Tech. Civil, Mechanical, ECE, EEE, Instrumentation and Chemical Engineering)

(Effective from the admitted batch of 2015–2016)

Time : Three hours Maximum : 70 marks

Answer ALL questions in Part A and FOUR questions from Part B.

All questions carry equal marks.

Questions of Part A must be answered at one place only.

PART A

1. (a) Find the unit normal to the level surface $x^2+y-z=4$ at $(2,0,0)$.

(b) Find the values of $div\bar{r}$ and $curl\bar{r}$, where $\bar{r}=x_i+y_j+z\bar{k}$.

(c) Define circulation.

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(d) Form a partial differential equation by eliminating the constants a and b from $z = ax^n + by^n$.

(e) What are the assumptions to be made in deriving the one dimentional wave equation?

(f) Write any two properties for Fourier transforms.

(g) Find the finite Fourier sine transform of $f(x) = 2x, 0 < x < 4$.

PART B

2. (a) If the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at the point $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to the z-axis, then find the values of a,b,c.

(b) If $\overline{R} = xi + yj + zk, r = |\overline{R}|$ and $\overline{A}$ is a constant vector then prove that

$$\nabla \times \left(\frac{\overline{A} \times \overline{R}}{r^n} \right) = \frac{2-n}{r^n} \overline{A} + n \frac{(\overline{A}.\overline{R})}{r^n + 2} \overline{R}.$$

3. (a) State Green's theorem and apply it to evaluate $\int_c \left[\left(2x^2 - y^2\right) dx + \left(x^2 + y^2\right) dy \right]$, where C is the boundary of the area enclosed by the x- axis and the upper-half of the circle $x^2 + y^2 = a^2$.

(b) By transforming to triple integral, evaluate $\iint_s \left(x^3 dy\, dz + x^2 y\, dz\, dx + x^2 z\, dx\, dy\right)$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discuss $z = a$ and $z = b$.

4. (a) Prove that a spherical polar coordinate system is orthogonal. Also express the vector field $xyi + yzj + zx\overline{k}$ in spherical polar coordinate system.

(b) Solve $z^2\left(p^2 + q^2\right) = x^2 + y^2$.

5. (a) Solve : $\left(D^3 + D^2 D' - DD'^2 - D'^3\right)z = e^x \cos 2y$.

(b) Solve : $\left(D^2 - D'^2 - 3D + 3D'\right)z = xy$.

6. (a) A tightly streched string with fixed end point $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = k \sin\dfrac{3\pi x}{l} \cos\dfrac{2\pi x}{l}$. If it is released from rest from this position, determine the displacement $y(x,t)$.

(b) Derive the solution of Laplace equation in Polar coordinates.

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Oct-16

8. (a) State convolution theorem and use it to evaluate :

$$Z^{-1}\left\{\frac{8z^2}{(2z-1)(4z+1)}\right\}.$$

8z

(b) Solve by using Z-transforms $y_{n+2}+4y_{n+1}+3y_n=3^n$, given that $y_0=0$ and $y_1=1$.

APRIL-15

OK 3131772029 20-04-2015 APRIL-15 EEE

II/IV B.E./B.Tech. DEGREE EXAMINATION.

Second Semester

Electronics and Communication Engineering

MATHEMATICS – IV

(Common for all branches)

(Effective from the admitted batch of 2013–2014)

Time : Three hours Maximum : 70 marks

Part A is compulsory.

Answer any FOUR questions from Part B.

Each question will carry 14 marks.

PART A

1. (a) Is $f(z)=z|z|$ is analytic at the origin. Justify.

(b) What kind of singularity does the function $f(z)=e^{1/z}$ has at $z=0$?

(c) Define critical points of the bilinear transformation.

(d) Write any two objectives of the sampling.

(e) Write any two properties of F-distribution.

(f) Solve : $y_{n+1} - 2y_n \cos\alpha + y_{n-1} = 0$

(g) Find the Z-transform of $f(n) = \{1, 2, 3, 4\}$. Also find the ROC.

PART B

2. (a) Find the harmonic conjugate of $u = e^{x^2-y^2}.\cos 2xy$. Hence find $f(z)$ in terms of z.

(b) If $f(z)$ is an analytic function then prove that

$$\nabla^2 |f(z)|^2 = 4|f'(z)|^2.$$

3. (a) Evaluate $\int_C \frac{z}{z^2+1}\, dz$, where $C: \left|z + \frac{1}{2}\right| = 2$.

(b) Expand $f(z) = \frac{e^{2z}}{(z-1)^3}$ about $z = 1$ as a Laurent's series. Also find the region of convergence.

4. (a) State residue theorem and use it to evaluate.

$$\int_C \frac{12z-7}{(2z+3)(z-1)^2}\, dz, \text{ where } C: x^2+y^2 = 4.$$

(b) Apply the calculus of residues to evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx.$$

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2015

5. (a) In a normal distribution : 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

(b) Experience had shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.

6. (a) The average breaking strength of the steel rod is specified to be 18.5 thousand pounds. To test this sample of 14 rods was tested. The mean and standard deviation obtained were 17.85 and 1.955 respectively. Is the result of experiment significance?

(b) A die was thrown 264 times with the following frequency results :

No. of appeared on the die :	1	2	3	4	5	6
Frequency:	40	32	28	58	54	52

Test whether the die is unbiased?

7. (a) Solve the difference equation :

$$u_{n+2} - 7u_{n+1} - 8u_n = n(n-1)2^n.$$

(b) If $Z(u_n) = \frac{z}{z-1} + \frac{z}{z^2+1}$ then find $Z(u_{n+2})$.