SIR C R REDDY COLLEGE OF ENGINEERING, ELURU

DEPARTMENT OF INFORMATION TECHNOLOGY

MATHEMATICS-III

HANDOUT

SUBJECT: MATHEMATICS-III

CLASS: II/IV B.Tech (A & B sections)Semester-I, A.Y.2023-2024

INSTRUCTORS: S.SIREESHA,D.ANUSHA

Course Handout Index

College Vision & Mission

Vision: To emerge as a premier institution in the field of technical education and research in the state and as a home for holistic development of the students and contribute to the advancement of society and the region.

Mission: To provide high quality technical education through a creative balance of academic and industry oriented learning; to create an inspiring environment of scholarship and research; to instill high levels of academic and professional discipline; and to establish standards that inculcate ethical and moral values that contribute to growth in career and development of society in general.

Department Vision & Mission

Vision: To be a premier department in the region in the field of Information Technology through academic excellence and research that enable graduates to meet the challenges of industry and society.

Mission: To Provide dynamic teaching-learning environment to make the students industry ready and advancement in career; to inculcate professional and leadership quality for better employability and entrepreneurship; to make high quality professional with moral and ethical values suitable for industry and society.

Program Educational Objectives (PEOs)

PEO1: Solve real world problems through effective professional skills in Information Technology industry and academic research.

PEO2: Analyze and develop applications in Information Technology domain and adapt to changing technology trends with continuous learning.

PEO3: Practice the profession in society with ethical and moral values.

Program Outcomes (POs)

PO1: Engineering Knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2: Problem Analysis: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using the first principles of mathematics, natural sciences, and engineering sciences.

PO3: Design/Development of Solutions: Design solutions for complex engineering problems and system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, society, and environmental considerations.

PO4: Conduct Investigations of Complex Problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5: Modern Tool Usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO6: The Engineer and Society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7: Environment and Sustainability: Understand the impact of the professional engineering solutions in society and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8: Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9: Individual and Team Work: Function effectively as an individual, and as a member or leader in diverse teams, and in multi-disciplinary settings.

PO10: Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11: Project Management and Finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multi-disciplinary environments.

PO12: Life-long Learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Specific Outcomes (PSOs)

PSO1: Design Skill: Design and develop softwares in the area of relevance under realistic constraints.

PSO2: New Technology: Adapt new and fast emerging technologies in the field of Information Technology.

JNTUK Academic Calendar

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Directorate of Academic Planning
JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY KAKINADA
KAKINADA-533003, Andhra Pradesh, INDIA
(Established by AP Government Act No. 30 of 2008)

Lr. No. DAP/AC/II Year /B. Tech/2023 *Date 01.08.2023*

Dr. KVSG Murali Krishna,
M.E, Ph.D.,
Director, Academics & Planning
JNTUK, Kakinada

To
All the Principals of Affiliated Colleges,
JNTUK, Kakinada.

Academic Calendar for II Year - B. Tech for the AY 2023-24

Description	From	To	Weeks
I SEMESTER			
Commencement of Class Work	**07.08.2023**		
I Unit of Instruction	07.08.2023	30.09.2023	8W
I Mid Examinations	25.09.2023	30.09.2023	
II Unit of Instructions	02.10.2023	25.11.2023	8W
II Mid Examinations	20.11.2023	25.11.2023	
Preparation & Practicals	27.11.2023	09.12.2023	2W
End Examinations	11.12.2023	23.12.2023	2W
Commencement of II Semester Class Work	**27.12.2023**		
II SEMESTER			
I Unit of Instructions	**27.12.2023**	17.02.2024	8W
I Mid Examinations	12.02.2024	17.02.2024	
II Unit of Instructions	19.02.2024	13.04.2024	8W
II Mid Examinations	08.04.2024	13.04.2024	
Preparation & Practicals	15.04.2024	27.04.2024	2W
End Examinations	29.04.2024	11.05.2024	2W
Summer Internship	13.05.2024	06.07.2024	8W
Commencement of III- I Class Work	08.07.2024		

Director
Academic Planning
JNTUK Kakinada
1/8/23

Director,
Academics & Planning,
JNTUK

Copy to the Secretary to the Hon'ble Vice Chancellor, JNTUK
Copy to Rector, JNTUK
Copy to Registrar, JNTUK
Copy to Director Academic Audit, JNTUK
Copy to Director of Evaluation, JNTUK

SIR C R REDDY COLLEGE OF ENGINEERING

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IQAC

DEPARTMENT OF INFORMATION TECHNOLOGY

II/IV ACADEMIC CALENDAR 2023 – 2024

EVENTS / ACTIVITIES	I- SEMESTER	II- SEMESTER
Registration of Credits/Electives	15-07-2023 to 5-07-2023	10-12-2023 To 24-12-2023
Commencement of classes	7-08-2023	27-12-2023
Class work – 1st Phase of Instruction (From… To…)	07-08-2023 To 30-09-2023	27-12-2023 To 17-02-2024
Class Review Committee Meeting-I/Parent-Teachers Meet	September 2023	February 2024
Guest Lecture/Seminar/Workshop	September 2023	February 2024
Assignment - I	10-09-2023	01-01-2023
MID Examination – I & Quiz - I	25-09-2023 To 30-09-2023	12-02-2024 To 17-02-2024
Mid-Semester Feedback	1-10-2023	18-02-2024
Last date for display of Marks/Answer Scripts	8-10-2023	25-02-2024
Class work – 2nd Phase of Instruction (From… To…)	02-10-2023 To 25-11-2023	19-02-2024 To 13-04-2024
Remedial classes	After 1st MID	After 1st MID
Class Review Committee Meeting-II	November 2023	April 2024
Guest Lecture/Seminar/Workshop	November 2023	March 2024
Assignment - II	01-11-2023	22-03-2024
MID Examination – II & Quiz - II	20-11-2023 To 25-11-2023	08-04-2024 To 13-04-2024
Class work last working day	18-11-2023	05-04-2024
End-Semester Feedback & Course End Survey	26-11-2023	14-04-2024
Last date for display of Marks/Answer Scripts	30-11-2023	21-04-2024
Preparation holidays and Semester End Practical Examinations	27-11-2023 To 09-12-2023	15-04-2024 To 27-04-2024
Semester End Theory Examinations	11-12-2023 To 23-12-2023	29-04-2024 To 11-05-2024
Summer Internship	--	13-05-2024 To 06-07-2024

HOD

HEAD OF THE DEPARTMENT
Information Technology
Sir C.R.R. College of Engg.
ELURU-534 007

PRINCIPAL

Principal
Sir C.R.R.College of Engineering
ELURU - 534 007

Course description:

Vector calculus includes both vector differentiation and vector integration concepts which describes differentiation problems and the problems of work done also volumes, areas respectively. Laplace transforms includes basic Laplace problems and its applications. Fourier series describes periodic functions and Fourier Transforms concept describes non-periodic and finite series problems. First order PDE includes the concepts of linear and non linear equations. Higher PDE describes the concepts of homogeneous and non-homogeneous DE and it's applications (wave equations.)

Scope and objectives:

- To familiarize the techniques in partial differential equations
- To furnish the learners with basic concepts and techniques at plus two level to lead them into advanced level by handling various real world applications.

Prerequisite:

Vector Calculus concepts used to solve differential and integration problems. From Laplace concepts, you will learn how to solve Laplace problems and applications. Using Fourier Series and Fourier transforms you can solve periodic functions and non periodic functions respectively. From PDE concepts you can solve first and higher ordinary partial differential equations and can solve physical applications in real life.

<u>Course Outcomes</u>

After the completion of the course, student will be able to

CO	CO Description	Level
CO1	Apply the concepts of vector calculus to the problems of work done by a force, circulation and flux	L3
CO2	Apply Laplace Transforms to solve the ordinary differential equations	L3
CO3	Compute Fourier series of the periodic function and Apply Fourier transform to a range of non-periodic function.	L3
CO4	Solve the first and higher ordinary partial differential equations and apply to various physical problems	L3

Syllabus

UNIT I: Vector calculus:(10 hrs)
Vector Differentiation: Gradient – Directional derivative – Divergence – Curl – Scalar Potential. Vector Integration: Line integral – Work done – Area – Surface and volume integrals – Vectorintegral theorems: Greens, Stokes and Gauss Divergence theorems (without proof).

UNIT II: Laplace Transforms:(10 hrs)
Laplace transforms of standard functions – Shifting theorems – Transforms of derivatives and integrals – Unit step function – Dirac's delta function – Inverse Laplace transforms – Convolutiontheorem (without proof).

Applications: Solving ordinary differential equations (initial value problems) using Laplace transforms.

UNIT III: Fourier series and Fourier Transforms:(10 hrs)

Fourier Series: Introduction – Periodic functions – Fourier series of periodic function – Dirichlet'sconditions – Even and odd functions – Change of interval – Half-range sine and cosine series.
Fourier Transforms: Fourier integral theorem (without proof) – Fourier sine and cosine integrals –Sine and cosine transforms – Properties – inverse transforms – Finite Fourier transforms.

UNIT IV: PDE of first order:(8 hrs)
Formation of partial differential equations by elimination of arbitrary constants and arbitrary functions – Solutions of first order linear (Lagrange) equation and nonlinear (standard types) equations.

UNIT V: Second order PDE and Applications:(10 hrs)

Second order PDE: Solutions of linear partial differential equations with constant coefficients RHS term of the type e^{ax+by}, $\sin(ax+by), \cos(ax+by), x^m y^n$.
Applications of PDE: Method of separation of Variables – Solution of One dimensional Wave,Heat and two- dimensional Laplace equation.

Text Books:

1) B. S. Grewal, Higher Engineering Mathematics, 43rd Edition, Khanna Publishers.
2) B. V. Ramana, Higher Engineering Mathematics, 2007 Edition, Tata Mc. Graw HillEducation.

Reference Books:

1) Erwin Kreyszig, Advanced Engineering Mathematics, 10th Edition, Wiley-India.
2) Dean. G. Duffy, Advanced Engineering Mathematics with MATLAB, 3rd Edition, CRCPress.
3) Peter O' Neil, Advanced Engineering Mathematics, Cengage.
4) Srimantha Pal, S C Bhunia, Engineering Mathematics, Oxford University Press.

Lesson Plan

Sl.N O:	Topic Covered	Co's	Teaching methodologies
1.	Vector Differentiation: Gradient – Directional derivative – Divergence – Curl – Scalar Potential.	CO1	BB
2.	Vector Integration: Line integral – Work done – Area – Surface and volume integrals – Vector integral theorems: Greens, Stokes and Gauss Divergence theorems (without proof)	CO1	BB
3.	Laplace transforms of standard functions – Shifting theorems – Transforms of derivatives and integrals – Unit step function – Dirac's delta function	CO1	BB
4.	Inverse Laplace transforms – Convolution theorem (without proof). Applications: Solving ordinary differential equations (initial value problems) using Laplace transforms.	CO1	BB
5.	Fourier expansions –Functions having points of Discontinuity Change of Interval- Odd and Even Functions- Expansions of Odd of Even periodic Functions- Half range series- Parseval's formulae (10 hrs) Fourier Series: Introduction – Periodic functions – Fourier series of periodic function – Dirichlet's conditions – Even and odd functions – Change of interval – Half-range sine and cosine series.	CO1	BB
6.	Fourier Transforms: Fourier integral theorem (without proof) – Fourier sine and cosine integrals –Sine and cosine transforms – Properties – inverse transforms – Finite Fourier transforms.	CO1	BB
7.	Formation of partial differential equations by elimination of arbitrary constants and arbitrary functions	CO1	BB
8.	Solutions of first order linear (Lagrange) equation and nonlinear (standard types) equations.	CO1	BB
9.	Second order PDE: Solutions of linear partial differential equations with constant coefficients – RHS term of the type eax+by ,sin(ax +by), cos(ax + by), xm yn	CO1	BB
10.	Applications of PDE: Method of separation of Variables – Solution of One dimensional Wave, Heat and two-dimensional Laplace equation	CO1	BB

S. No	Components	Internal	External	Total
1	Theory	30	70	100
2	Engineering Graphics/Design/Drawing	30	70	100
3	Practical	15	35	50
4	Mini Project/Internship/Industrial Training/ Skill Development programmes/Research Project	-	50	50
5	Project Work	60	140	200

Marks Range Theory (Max – 100)	**Marks Range Lab (Max – 50)**	**Level**	**Letter Grade**	**Grade Point**
≥ 90	≥ 45	Outstanding	A+	10
≥80 to <89	≥40 to <44	Excellent	A	9
≥70 to <79	≥35 to <39	Very Good	B	8
≥60 to <69	≥30 to <34	Good	C	7
≥50 to <59	≥25 to <29	Fair	D	6
≥40 to <49	≥20 to <24	Satisfactory	E	5
<40	<20	Fail	F	0
-		Absent	AB	0

Timetable

Day/Time	**09.00-09.50**	**09.50-10.40**	**11.00-11.50**	**11.50-12.40**	**01.40-02.30**	**02.30-03.20**	**03.20-04.10**	**04.10-05.00**
Mon	A		B					
Tue					B			
Wed	B					A		
Thu			A					
Fri	A		B		B			
Sat			A		*********			

UNIT WISE QUESTIONS

UNIT-I

Vector Calculus **Total hours: 18**

Gradient (01 hour)

1. Find $\nabla\phi$, where $\phi(x, y, z) = \log(x^2 + y^2 + z^2)$
2. Prove that $\nabla(r^n) = n\, r^{n-1}\bar{r}$

Unit normal, Directional Derivative and Angle between two surfaces (03 hours)

1. Find the unit normal to the surface $xy + yz + zx = 3$ at point (1,1,1)
2. In what direction from the point $(1, -2, -1)$ the directional derivation of $\phi = x^2yz + 4xz^2$ is maximum? What is the magnitude of this maximum? **(2016)**
3. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\bar{i} - \bar{j} - 2\bar{k}$. **(2016, 2019)**
4. Find the directional derivative of $\phi = xy + yz + zx$ at A in the directional of $\overline{AB}$ the where $A(1,2,-1)$ and $B(1,2,3)$ (**2023**)
5. Find the directional derivative of the function $\phi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x\, logz - y^2 + 4 = 0$ at the point $(-1,2,1)$ **(2011, 2015)**
6. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$**(2014, 2015, 2016)**
7. Find the angle between the surfaces $xy = z^2$ at the point (4,1,2) and $(3,3,-3)$**(2015)**

Divergence, Curl, Solenoidal, irrotational and orthogonal (04 hours)

1. Show that the vector $(x^2 - yz))\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. **(2013, 2014, 2016)**
2. Find the constants a, b, c so that the vector $\bar{A} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + cy + 2z)\bar{k}$is irrotational. Also find ϕ such that $\bar{A} = \nabla\phi$. **(2016, 2015, 2019)**
3. (i) Prove that $r^n\bar{r}$ is Solenoidal if $n = -3$. **(2015)** (ii) Prove that $\frac{\bar{r}}{r^3}$ is Solenoidal. **(2015)**
4. Show that $\bar{F} = (2xy + z^3)\bar{i} + x^2\bar{j} + 3xz^2\bar{k}$ is conservative force field and find the scalar potential. **(2010)**
5. Find a, b such that $ax^2 - byx = (a + z)$ and $4ax^2y + z^3 = 4$ cut orthogonally at$(1,1,-2)$. **(2019)**
6. Prove that (i) $\nabla^2(r^n) = n(n+1)r^{n-2}$**(2009, 2019)** (ii) $\nabla^2(logr) = \frac{2}{r^2}$ **(2015)**
   (iii) $\nabla\left(\nabla.\frac{\bar{r}}{r}\right) = -\frac{2}{r^3}\bar{r}$ **(2015)** (iv) $\nabla.\left(r\,\nabla\left(\frac{1}{r^3}\right)\right) = \frac{3}{r^4}$ **(2015)**
7. Show that $\nabla\phi$ is both Solenoidal and irrotational if $\nabla^2\phi = 0$. **(2015)**
8. If ϕ and ψ are scalar function, then prove that $\nabla\phi \times \nabla\psi$ is Solenoidal. **(2015)**
9. Prove that (i) $\nabla.\frac{\bar{r}}{r^3} = 0$**(2008, 2009)** (ii) $\nabla^2\left(\frac{1}{r}\right) = 0$
10. Determine the constant a, if $\bar{F} = \frac{1}{x^2+y^2}\left(x\bar{i} + ay\bar{j}\right) + \bar{k}$is Solenoidal. **(2023)**
11. Prove that $\nabla.\left(\bar{A} \times \bar{B}\right) = \bar{B}.\left(\nabla \times \bar{A}\right) - \bar{A}.\left(\nabla \times \bar{B}\right)$ **(2014, 2016)**
12. Prove that $\nabla \times \left(\bar{A} \times \bar{B}\right) = \left(\nabla.\bar{B}\right)\bar{A} - \left(\nabla.\bar{A}\right)\bar{B} + \left(\bar{B}.\nabla\right)\bar{A} - \left(\bar{A}.\nabla\right)\bar{B}$- **(2014, 2016)**
13. Prove that $\nabla \times \nabla \times \nabla \times \nabla \times \bar{F} = \nabla^4\bar{F}$, if $\bar{F}$ is solenoidal.

Line Integral, Surface Integral and Volume Integral (06 hours)

1. Find the total work done by force $\overline{F} = 2xy\overline{i} - 4Z\,\overline{j} + 5x\overline{k}$ along the curve $x = t^2, y = 2t + 1, z = t^3$ for $t = 1, t = 2$. **(2015)**
2. Evaluate $\int_c \overline{F}.\overline{dr}$ *where* $\overline{F} = 2x^2yz\overline{i} + x^2y\,\overline{j}$ and C is the curve $x = t, y = t^2, z = t^3$ from $t = 0$ to $t = 1$ **(2023)**
3. Find the work done in moving particle in the force field $\overline{F} = 2x^2\overline{i} + (2yz - x)\overline{j} - y\overline{k}$ along
   (i) the straight line (0,0,0) to (3,1,2)
   (ii) the space curve $x = 3t^2, y = t, z = 3t^2 - t$ from $t = 0$ to 1. **(2015)**
4. Show that $\overline{F} = (2xy + z^2)\overline{i} + x^2\overline{j} + 3xz^2\overline{k}$ is conservative force field and find its potential function and also find work done in moving an object in this field (1,-2,1) to (3,1 4) **(2007)**
5. Evaluate $\int_s \overline{F}.\overline{n}\,ds$ where $\overline{F} = 18z\overline{i} - 12\,\overline{j} + 3y\,\overline{\mathrm{k}}$ and S is part of the plane $2x + 3y + 6z = 12$. Located in first octant. **(2023)**
6. Evaluate $\int_s \overline{F}.\overline{n}\,ds$ where $\overline{F} = z\overline{i} + x\,\overline{j} - 3y^2z\,\overline{\mathrm{k}}$ where S is the surface of the cylinder $x^2 + y^2 = 1$ in the first octant between $z = 0$ and $z = 2$ **(2008, 2011)**
7. If $\overline{F} = x^3\overline{i} + y^3\overline{j} + z^3\overline{k}$, evaluate $\int_v div\,\overline{F}\,dv$, where v is the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$
8. If $\overline{V}$ is the first octant bounded by $y^2 + z^2 = 9$ and the plane $x = 2$ and $\overline{F} = 2x^2y\overline{i} - y^2\overline{j} + 4xz^2\overline{k}$.

   Then evaluate $\iint_s \overline{F}.\overline{n}\,ds$. **(2004, 2006, 2016)**

Green's, Stoke's and Gauss divergence theorems (06 hours)

1. Evaluate $\oint_c (2xy - x^2)dx + (x + y^2)dy$ where C is the closed in xy – plane bounded by the curves $y = x^2$ and $y^2 = x$. **(2015, 2019)**
2. Verify Green's theorem for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region enclosed by the lines $x = 0, y = 0$ and $x + y = 1$ **(2003, 2007)**
3. Verify Green's theorem in plane for $\oint_c (x^2 - xy^3)dx + (y^2 - 2xy)dy$, where C is the square with $(0,0), (2,0), (2,2)$ and $(0,2)$. **(2008, 2009)**
4. Evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ using Green's theorem where C is the boundary of the surface in the xy – plane enclosed by x – axis and semi-circle $x^2 + y^2 = a^2$(**2015**)
5. Evaluate $\oint_c (e^x\,dx + 2y\,dy - dz)$, where C is the curve $x^2 + y^2 = 9$ and $z = 2$.(**2023**)
6. Evaluate $\iint_s (curl\,\overline{A}\,.\overline{n})\,ds$, where $\overline{A} = y\overline{i} + (x - 2z)\overline{j} - xy\overline{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ about xy – plane. (**2014**)
7. Evaluate $\oint_c (y - z + 2)dx + (yz + 4)dy - xz\,dz$ over the surface of the cube $x = 0, x = 2, y = 0, y = 2$ and $z = 0, z = 2$ above the xy – plane. (**2006, 2011, 2013**)
8. Verify Stokes's theorem for $\overline{F} = y\overline{i} + z\,\overline{j} + x\,\overline{k}$ for the upper half of the sphere $x^2 + y^2 + z^2 = 1$.
9. Verify Stokes's theorem for $\overline{F} = (x^2 + y^2)\overline{i} - 2xy\,\overline{j}$ taken around the rectangle bounded the lines $x = \pm\,a, y = 0, y = b$ **(2023)**

10. Verify Stokes's theorem for $\overline{F} = (2x - y)\overline{i} - yz^2\overline{j} - y^2z\,\overline{k}$ over the half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the projection of the xy – plane. **(2023)**
11. Verify Gauss divergence theorem for $\overline{F} = 4x\overline{i} - y^2\overline{j} + xz\,\overline{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1$ and $z = 0, z = 1$ (**2007, 2014**)
12. If $\overline{V}$ is the first octant bounded by $y^2 + z^2 = 9$ and the plane $x = 2$ and for $\overline{F} = 2x^2y\overline{i} - y^2\overline{j} + 4xz^2\overline{k}$. Then evaluate $\iint_s \overline{F}.\overline{n}\,ds$. (**2004, 2006, 2016**)
13. Use Gauss divergence theorem, evaluate for $\iint_s (yz^2\overline{i} + zx^2\overline{j} + 2z^2\overline{k)}\,.\,ds$ where S is the closed surface bounded by the xy – plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane (**2019**)
14. Evaluate $\iint_s x^3dydz + x^2ydxdz + x^2zdxdy$ over the surface bounded by the planes $z = 0, z = b$ and the cylinder $x^2 + y^2 = a^2$
15. Using Divergence theorem, evaluate $\iint_s \overline{F}.\overline{n}\,ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = b^2$ in the first octant where $\overline{F} = y\overline{i} + z\,\overline{j} + x\,\overline{k}$ **(2017)**

UNIT-II

LAPLACE TRANSFORMS **Total Hours: 15**

Basic Problems: (2hrs)

Find the Laplace Transform of

(i)$e^{2t}+4t^3-2sin3t+3cos3t$**(2003)**

(ii)$3cosh5t-4sinht$**(2006)**

1. Find the Laplace Transform of $\sin 2t\ sin3t$**(2016)**
2. Show that the function $f(t)=t^3$ is of exponential order and find its Laplace Transform. **(2011,2018)**
3. Find the Laplace Transform of following

   (i)$f(t)=\begin{cases}\cos t & 0<t<\pi \\ \sin t & t>\pi\end{cases}$**(2013)**

   (ii) $e^{-3t}\ (2cos5t-3sin5t)$**(2010, 2012)**
4. Find (i)$L\{(t+3)^3e^{2t}\}$ (ii)$L\{e^{3t}\sin^2 t\}$ (iii)$L\{\sqrt{t}e^{3t}\}$**(2012,2016,2018,)**
5. Find the Laplace Transform of following $g(t)=\begin{cases}\cos\left(t-\frac{2\pi}{3}\right) & if\ t>\frac{2\pi}{3} \\ 0 & if\ t<\frac{2\pi}{3}\end{cases}$ (2010,2016)
6. Find the Laplace Transform of $(sint-cost)^3$**(2023)**

Multiplication by t: (1hrs)

1. Find $(i)\mathrm{L}\{3cos4(t-2)u(t-2)\}(ii)\mathrm{L}\{\mathrm{t}\ sin3\mathrm{t}\ cos2\mathrm{t}\}$ $(iii)L\{t^2e^{-2t}\}$**(2010,2015,2018)**
2. Find $L\{te^{-t}sin2\mathrm{t}\}$**(2006,2015)**

Division by t: (1hrs)

1. Find (i) $\mathrm{L}\left\{\frac{\mathrm{e}^{-\mathrm{at}}-\mathrm{e}^{-\mathrm{bt}}}{\mathrm{t}}\right\}$ (ii) $\mathrm{L}\left\{\frac{\sin 3\mathrm{t}cost}{\mathrm{t}}\right\}$**(2010,2012,2015)**
2. Find the Laplace Transform of $\mathrm{f(t)}=\frac{\mathrm{cosat}-cos\mathrm{bt}}{\mathrm{t}}$**(2014,2015)**
3. Evaluate $\mathrm{L}\left\{\frac{1-cost}{\mathrm{t}^2}\right\}$**(2015)**

Integrals (0 to t): (1hrs)

1. Find (i) $\mathrm{L}\left\{\int_0^{\mathrm{t}} \mathrm{e}^{-\mathrm{t}}\mathrm{cost}dt\right\}$ (ii) $\mathrm{L}\left\{\int_0^{\mathrm{t}} \mathrm{te}^{-\mathrm{t}}\mathrm{sin2t}dt\right\}$

   (iii) $\mathrm{L}\left\{\int_0^{\mathrm{t}}\int_0^{\mathrm{t}}\int_0^{\mathrm{t}} \frac{t}{2}e^{2t}t^2dtdtdt\right\}$**(2015,2009,2023)**

Evaluate Integrals: (1 hrs)

1. Using Laplace Transform, Evaluate $\int_0^\infty t\ \mathrm{e}^{-\mathrm{t}}\mathrm{sint}dt$**(2012,2013)**
2. Using Laplace Transform, Evaluate $\int_0^\infty \frac{e^{-t}-e^{-2t}}{\mathrm{t}}dt$**(2010,2011,2012,2023)**
3. Using Laplace Transform, Evaluate $\int_0^\infty \frac{\mathrm{cosat}-cos\mathrm{bt}}{\mathrm{t}}dt$**(2013,2017)**
4. Using Laplace Transform, Evaluate $\int_0^\infty \frac{e^{-t}\sin^2 t}{\mathrm{t}}dt$**(2012,2016,2019)**
5. Using Laplace Transform, Evaluate $\int_0^\infty \mathrm{t}^3\ \mathrm{e}^{-\mathrm{t}}\mathrm{sint}dt$
6. **Periodic Funcions: (1hr)**
7. 1. Find $\mathrm{L}\{\mathrm{f(t)}\}$ where $\mathrm{f(t)}$ is a periodic function of period 2π and it is given
8. by $f(t)=\begin{cases}sint & 0<t<\pi \\ 0 & \pi<t<2\pi\end{cases}$**(2022)**

Theorems: (2hrs)

1. If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\{e^{at}f(t)\} = \bar{f}(s-a)$, s-a>0
2. If $L\{f(t)\} = \bar{f}(s)$ and $u(t-a) = \begin{cases} 0 & if\ t < a \\ 1 & if\ t > a \end{cases}$ then prove that $L\{u(t-a)\} = \frac{e^{-as}}{s}$**(2016,2018)**
3. If $L\{f(t)\} = \bar{f}(s)$ and $g(t) = \begin{cases} f(t-a) & if\ t > a \\ 0 & if\ t < a \end{cases}$ then prove that $L\{g(t)\} = e^{-as}\bar{f}(s)$**(2011)**
4. If $f(t)$ is continuous and of exponential order and $f'(t)$ is sectionally continuous then prove that Laplace Transform of $f'(t)$ is given by
   $L\{f'(t)\} = s\bar{f}(s) - f(0)$**(2011)**
5. If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left\{\int_0^t f(u)du\right\} = \frac{1}{s}\bar{f}(s)$. **(2022)**
6. If $f(t)$ is continuous and of exponential order and $L\{f(t)\} = \bar{f}(s)$ then prove that
   $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\bar{f}(s)\right]$**(2012)**
7. If $L\{f(t)\} = \bar{f}(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \bar{f}(s)ds$**(2011)**
8. Find Laplace Transform of unit impulse function or dirac delta function. **(2015,2016,2022)**

Inverse Laplace

Basic problems: (2 hrs)

1. Find $L^{-1}\left\{\frac{3(s^2-2)^2}{2s^5}\right\}$**(2007)**
2. Find $L^{-1}\left\{\frac{4}{(s+1)(s+2)}\right\}$**(2008)**
3. Find $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+25)}\right\}$**(2008,2012)**
4. Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+9)(s^2+25)}\right\}$**(2010)**
5. Find $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$**(2010)**
6. Find $L^{-1}\left\{\frac{1}{(s+1)^3}\right\}$**(2012,2019)**
7. Find $L^{-1}\left\{\frac{s}{(s^4+4a^4)}\right\}$**(2012,2015)**
8. Find $L^{-1}\left\{\frac{1+e^{-\pi s}}{s^2+1}\right\}$**(2009)**
9. Find the Inverse Laplace Transform of $\log\left(\frac{s+1}{s-1}\right)$**(2014,2018)**
10. 10.Find (i) $L^{-1}\left\{\frac{s+3}{(s^2+6s+13)^2}\right\}$**(2011)**
    (ii) $L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}$**(2013)**
    (iii) $L^{-1}\left\{cot^{-1}\left(\frac{s+2}{3}\right)\right\}$(2012) (iv) $L^{-1}\left\{\frac{2s^2-6s+5}{(s^3-6s^2+11s-6)^2}\right\}$**(2011)**

11.Find the Inverse Laplace Transform of $\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}$**(2023)**

12. Find Inverse Laplace Transform of $\frac{s+5}{(s-1)^2(s+2)}$**(2023)**

13. Show that L{tsinat}= $\frac{2as}{(s^2+a^2)^2}$

Convolution Problems: (2hrs)

1. Using Convolution theorem, find

(i)$L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$**(2010,2022)**

(ii)$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$**(2014)**

(iii) $L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$**(2015)**

(iv)$L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$**(2022)**

2. State convolution theorem and use it to evaluate $L^{-1}\left\{\frac{1}{(s^2+4s+13)^2}\right\}$**(2016)**

Applications: (2hrs)

1. Using Laplace Transform solve $(D^2+4D+5)y = 5$ given that $y(0)=0$ $y'(0)=0$**(2012)**
2. Solve the differential equation $\frac{d^2x}{dt^2}+9x = \sin t$ using Laplace transform given that $x(0)=1$ $x(\frac{\pi}{2})=1$**(2012,2023)**
3. Using Laplace Transform solve the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y = e^{-t}\sin t$ given that $y(0)=0, y'(0)=1$**(2010,2014)**
4. Solve the D.E $y''+n^2y = a\sin(nt+2)$ $y(0)=0$ $y'(0)=0$ using Laplace Transform **(2010,2012,2023)**
5. Solve the D.E$(D^2+2D+1)x = 3te^{-t}$ if $y(0)=4$ $y'(0)=2$ using Laplace Transform**.(2018)**
6. Solve $y''-8y'+15y = 9te^{2t}$ $y(0)=5$ *and* $y'(0)=10$ using Laplace Transform**(2014,2023,2022)**

UNIT-III

Fourier Series and Fourier Transforms **Total Hours: 12**

Periodic function (2h)

1. Express $f(x) = x - \pi$ as forier series in the interval $-\pi < x < \pi$ (2011)
2. If $f(x) = \frac{(\pi-x)^2}{4}$ in the interval (0, 2 π). Show that
   $f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}$ And hence deduce that
   $$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$
   (2010, 2011, 2018, 2004, 2012)
3. Obtain the Fourier series for the function f(x)=x sinx, 0<x<2 π
   (2004,2006,2011,2013,2015)
4. Obtain the Fourier series for the function f(x)=x cosx , 0<x<2 π
   (2008,2009,2010,2013,2005,2016)
5. Find the Fourier series of period 2 π for the function $f(x) = x^2 - x$ in $(-\pi, \pi)$hence deduce the sum of the series$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$(2009, 2015)

6. Find the Fourier series for the function $f(x) = e^x$ in the interval (0, 2 π)(2016)

Functions having points of discontinuity(2h)

1. find the Fourier series of $f(x) = \begin{cases} 0, & for\ -\pi < x < 0 \\ x^2, & for\ 0 < x < \pi \end{cases}$ (2010,2011,2013,2018)

2. Find the Fourier series of $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & for\ -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & for\ 0 \le x \le \pi \end{cases}$.

Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2015)

3. Find the Fourier series to represent the function f(x) given by

$f(x) = \begin{cases} x, & for\ 0 \le x \le \pi \\ 2\pi - x, & for\ \pi \le x \le 2\pi \end{cases}$ Hence deduce that

$\frac{1}{1^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2012, 2015)

4. Find the Fourier series of $f(x) = \begin{cases} \frac{-(\pi+x)}{2}, & for\ -\pi \le x \le 0 \\ \frac{(\pi-x)}{2}, & for\ 0 \le x \le \pi \end{cases}$ (2010, 2011)

5. The intensity of an alternating current after passing through a rectifier is given by
$i(x) = \begin{cases} I_0 \sin x, & for\ 0 \le x < \pi \\ 0, & for\ \pi \le x \le 2\pi \end{cases}$ where I_0 is maximum current and the period is 2π. Express $i(x)$ as a Fourier series. (2002, 2005, 2006)

6. Find the Fourier series for $f(x) = \begin{cases} x, & for\ -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0, & for\ \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2019)

Even and odd functions(2h)

1. Expand the function $f(x) = x^2$ as a Fourier series in [$-\pi$, π]

   Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$ (2008, 2010, 2012)

   ii)$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{6}$ (2008)

   iii)$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2003, 2012)

2. Find the Fourier series for the function $f(x) = |x|$ in $-\pi < x < \pi$ and

   deduce that$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$ (2003,2014,2016,2019)

3. Obtain a Fourier expansion for $\sqrt{1-\cos(x)}$ in the interval $-\pi \leq x \leq \pi$ (2007)

4. Find the Fourier series to represent the function $f(x) = |\cos x|$,in $-\pi < x < \pi$ (2012)

5. Show that for $-\pi < x < \pi$,

   $\sin(ax) = \frac{2\sin(a\pi)}{\pi}\left[\frac{\sin x}{1^2-a^2} - \frac{2\sin 2x}{2^2-a^2} + \frac{3\sin 3x}{3^2-a^2} - \ldots\ldots\right]$(2004, 2005)

Half range Fourier series(1h)

1. Find the half range sine series for f(x) =x$(\pi - x)$ in 0<x< π

   Deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \ldots\ldots = \frac{\pi^3}{32}$ (2008, 2014, 2015)

2. Find cosine and sine series for f(x) =$(\pi - x)$ in [0, π] (2010, 2016)

3. Obtain the half-range sine and cosine series for the function

   $f(x) = \frac{\pi x}{8}(\pi - x)$inthe range 0≤x<≤ π

4. Find the Fourier sine and cosine series of

   $$f(x) = \begin{cases} x, & for\ 0 < x < \frac{\pi}{2} \\ \pi - x, & for\ \frac{\pi}{2} < x < \pi \end{cases}$$ (2012, 2015)

Chance of interval [-l, l](1h)

1. Find the Fourier series of the function $f(x) = e^x$ in the interval (0, 2).(2016)
2. find the Fourierseries to represent $f(x) = x^2 - 2$ when -2<x<2 (2003, 2005,2007, 2012,2013)
3. Find the Fourier series expansion for f(x) if $f(x) = \begin{cases} 2, & if\ -2 \leq x \leq 0 \\ x, & if\ o < x < 2 \end{cases}$ (2006, 2018)

Half range expansions(1h)

1. Obtain the half range cosine & sine series for f(x)=x in the interval (0, l) (2011)
2. Find the half range cosine series for f(x)=x(2-x) in $0 \leq x \leq 2$.

   And hence find sum of the series$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots$ (2002, 2003, 2004, 2011, 2013)

3. Find the half range sine series for$f(x) = \begin{cases} \frac{1}{4} - x, & if\ 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & if\ \frac{1}{2} < x < 1 \end{cases}$ (2012, 2019)

Fourier transforms

Fourier integrals(1h)

1. Using Fourier integral show that $e^{-ax} - e^{-bx} = \frac{2(b^2-a^2)}{\pi}\int_0^\infty \frac{\lambda \sin \lambda x}{(\lambda^2+a^2)(\lambda^2+b^2)} d\lambda$ (2006, 2007)
2. Using Fourier integral show that

$e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos \lambda x}{(\lambda^2+a^2)} d\lambda \; (a > 0,\; x \geq 0)$ (2014)

3. Using Fourier integral show that

$\int_0^\infty \frac{1-\cos \lambda\pi}{\lambda} \sin\lambda x \, d\lambda = \begin{cases} \frac{\pi}{2}, & if\ 0 < x < \pi \\ 0, & if\ x > \pi \end{cases}$ (2006, 2018)

4. Using Fourier integral show that $e^{-x}\cos x = \frac{2}{\pi}\int_0^\infty \frac{\lambda^2+2}{\lambda^2+4}\cos\lambda x \, d\lambda$ (2008, 2012, 2018)

Transforms(2h)

1. Find the Fourier transform of f(x) defined by $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin p}{p} dp$ or $\int_0^\infty \frac{\sin x}{x} dx$ and $\int_{-\infty}^\infty \frac{\sin a p \cos px}{p} dp$ (2003, 2004, 2011, 2012)
2. Find the Fourier transform of f(x) defined by $f(x) = \begin{cases} (1-x^2), & if |x| \leq 1 \\ 0, & if |x| > 1 \end{cases}$ (2014, 2015, 2018, 2019)and hence evaluate
   i) $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2} dx$ (2003, 2005, 2007, 2012)
   ii) $\int_0^\infty \frac{\sin x - x\cos x}{x^3} dx = \frac{\pi}{4}$ (2018)
3. Find the Fouriercosine transform of the function

$f(x) = \begin{cases} \sin ax, & if x < a \\ 0, & if x > a \end{cases}$ (2019)

4. Find the Fourier sine transform of $\frac{x}{a^2+x^2}$ (2002, 2004, 2005)
5. Find the Fourier sine &cosine transform of e^{-ax} a>0

Hence deduce the inverse formula (or) deduce the integrals
   i) $\int_0^\infty \frac{\cos px}{a^2+p^2} dp$
   ii) $\int_0^\infty \frac{p \sin px}{a^2+p^2} dp$ (2002, 2004, 2005, 2008, 2012, 2015)
6. Find the inverse Fourier sine transform of $F_s(p) = \frac{p}{1+p^2}$

(or) find f(x) if its Fourier sine transform is $\frac{p}{1+p^2}$ (2012, 2014, 2016)

7. Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}, \; -\infty < x < \infty$

(or) S.T the Fourier transform of $e^{-\frac{x^2}{2}}$ is reciprocal (2002, 2004,2008,2012,2015,2016, 2018)

8. Find the Fourier sine & cosine transforms of $f(x) = \frac{e^{-ax}}{x}$ and

Deduce that $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} \sin sx \, dx = \tan^{-1}\left(\frac{s}{a}\right) - \tan^{-1}\left(\frac{s}{b}\right)$ (2006, 2009, 2011, 2012, 2016, 2018)

UNIT-IV

FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS **Total Hours: 11**

I. Formation of P.D.E By Eliminating Arbitrary Constants (1 h)

1. Form the P.D.E by eliminating arbitrary constant from $z = ax + by + \left(\frac{a}{b}\right) - b$ **(Jan 2023)**
2. Form the P.D.E by eliminating arbitrary constant from $z = alog\left[\frac{b(y-1)}{1-x}\right]$ **(Aug 2022)**
3. Form the P.D.E by eliminating arbitrary constant a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ **(Aug2022)**
4. Form the P.D.E by eliminating arbitrary constants from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ **(Feb/Mar 2022)**
5. Form the P.D.E by eliminating arbitrary constants from $z = ax + by + a^2 + b^2$ **(July 2022)**
6. Form the P.D.E by eliminating arbitrary constants from $z = (x^2 + a)(y^2 + b)$ **(Jan 2023**)
7. Form the P.D.E by eliminating arbitrary constants from $z = xy + y\sqrt{x + a} + b$.**(Aug 2022)**

II. Formation Of P.D.E By Eliminating Arbitrary Functions From The Following : (2 h)

1. Form the P.D.E by eliminating arbitrary function from $z = f(x) + e^y g(x)$. **(Feb/Mar 2022)**
2. Form the P.D.E by eliminating arbitrary function f from the relation $z = x^2 + 2f\left(\frac{1}{y} + logx\right)$ **(July 2022).**
3. Form the P.D.E by eliminating arbitrary function from $ax + by + cz = \emptyset(x^2 + y^2 + z^2)$ **(Aug 2022)**
4. Form the P.D.E by eliminating arbitrary function from $\emptyset(x + y + z, x^2 + y^2 - z^2) = 0$ **(Jan 2023)**
5. Form the P.D.E by eliminating arbitrary function from $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0.$ **(Aug 2022)**
6. Form the P.D.E by eliminating arbitrary function from $\emptyset\left(\frac{y}{x}, x^2 + y^2 + z^2\right) = 0.$ **(Jul 2011**).
7. Form the P.D.E by eliminating arbitrary function from $z = yf(x^2 + z^2).$ **(Aug 2015**).

III. Solution Of Linear Partial Differential Equation : (2 h)

Lagrange's Linear Equation:

(i) *Method of Grouping :*

1. Solve $yzp + 2xq = xy$.
2. Solve $ptanx + qtany = tanz$ (**July 2022**)
3. Solve $\frac{y^2z}{x}p + xzq = y^2$ **(Sep 2014)**
4. Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ **(Feb 2015).**
5. Solve $p - q = \log(x + y)$.

(ii) *Method of Multipliers* : (2 h)

1. Solve$x(y-z)p+y(z-x)q=z(x-y)$. (**Aug 2022**).
2. Solve $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$. (**Aug 2022**)
3. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$. (**July 2022**)
4. Solve $(mz-ny)p+(nx-lz)q=ly-mx$. **(2015, Aug 2022)**
5. Solve $(x+2z)p+(4z-y)q=2x+y$. **(Aug 2022)**
6. Solve $\left(\frac{b-c}{a}\right)yzp+\left(\frac{c-a}{b}\right)xzq=\left(\frac{a-b}{c}\right)xy$. **(2015).**
7. Solve $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$**(2016, 2011**).

iii) Method of Grouping & Multipliers: (2 h)

1. Solve $xp-yq=y^2-x^2$ **(2014, 2015)**
2. Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$. (**Feb/March2022**)
3. Solve $(x^2-y^2-z^2)p+2xyq=2zx$. (**Feb/March2022**).
4. Solve $x^2p-y^2q=z(x-y)$. **(2012, 2016).**
5. Solve $(x^3+3xy^2)p+(y^3+3x^2y)q=2z(x^2+y^2)$.**(Jan 2023).**

<u>IV. Non Linear Partial Differential Equations :(Standard Types) (2 h)</u>

Standard Type: I - Equations of the form $\mathrm{f(p,q=0)}$

1. Solve $p^2+q^2=npq$. (**Jan 2023**)
2. Solve $\frac{1}{p}+\frac{1}{q}=1$ **(Dec 2016)**

Standard Type: II - Equations of the form $\mathrm{f(z,p,q)=0}$.

1. Solve $z^2=1+p^2+q^2$ **(2014, 2015, Feb/March 2022)**
2. Solve $p^2+pq=z^2$ (**2014, Aug 2022**)
3. Solve $\frac{p^2}{z^2}=1-pq$.**(Aug 2022**)
4. Solve $z^2(p^2+q^2+1)=1$ (**Dec 2015)**

Standard Type –III - Equations of the form $\mathrm{f(x,p)=g(y,q)}$

1. Solve $p^{\frac{1}{3}}-q^{\frac{1}{3}}=3x-2y$. (**Aug 2022**)
2. Solve$\left(\frac{p}{2}+x\right)^2+\left(\frac{q}{2}+y\right)^2=1$. (**2016**)
3. Find the complete integral of $pe^y=qe^x$. (**July 2022)**
4. Solve$p^2+q^2=x^2+y^2$. (**Feb/March 2022)**
5. Solve $\sqrt{p}+\sqrt{q}=2x-2y$. (**2015**).

Standard Type-IV – Equations of the form $\mathrm{z=px+qy+f(p,q)}$ (Clairaut's form)

1. Solve $(p-q)(z-xp-yq)=1$.**(2016, Aug 2023**)
2. Solve $pqz=p^2(xq+p^2)+q^2(yp+q^2)$ (**2015, 2014**)
3. Solve $z=px+qy+pq$ (**2015**)

Non linear Partial Differential Equations (Reducible forms)

1. Solve $x^2p^2+y^2q^2=z^2$**(2012, 2014**)
2. Solve $2x^4p^2-yzq-3z^2=0$ **(2015)**
3. Solve $z^2(p^2+q^2)=x^2+y^2$ **(2011, 2012, 2016, Jan 2023)**
4. Solve $z(p^2-q^2)=x-y$. (**Aug 2015)**
5. Solve $(x+pz)^2+(y+qz)^2=1$ **(2012).**

UNIT-V

Second and Higher Order Partial Differential Equations **Total Hours: 12**

Solving Homogenous Linear Equations with Constant Coefficients

1.Solve $(D^2 - 4DD' + 4D'^2)$ Z= 0 (Dec2018)

Rules for finding P.I. (Particular Integral)

Let the given PDE be $f(D, D')$ z = $F(x, y)$

Case 1: When $F(x, y) = e^{ax+by}$

1.Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$ (Sep2014,Aug2022)

2.Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ (Nov2015,2018,July2022)

3.Solve $(4D^2 + 12DD' + 9D'^2)$ Z = e^{3x-2y} (Feb2014,Oct2018,May2019)

Case 2: When $F(x, y)$ = Sin(ax+by) or Cos(ax+by)

1. Solve $(D^3 - 4D^2D' + 4DD'^2)$ Z = 2 Sin(3x+2y) (Dec2016,Jan2023,Aug2022)
2. Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2}$ = Sin(2x+3y) (Feb2022)
3. Solve$(D^2 - DD')$ Z = Sin x cos2y (Dec2017,May2019)
4. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y}$ = cos x cos2y (July2022)

5.Solve $(D^3 - 7DD'^2 - 6D'^3)$ Z = Sin(x+2y) + e^{2x+y} (Dec2016)

6. Solve $(D^2 - D'^2)$ Z = cos (x+y) (Dec2017)

Case 3: When $F(x, y) = x^m y^n$

1. Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y}$ = 2 e^{2x} + $3x^2y$ (Feb2022)
2. Solve $(D^2 + DD' - 6D'^2)$ Z = x + y (Aug2022)

3.Solve $(D^3 - D'^3)$ Z = x^3y^3 (Aug2022)

4. Solve $(D^2 - D'^2)$ Z = $x^2 + y^2$ (Aug2022)

Case 4: In case of any function of or when solution fails for any case by above given methods

P.I = $\frac{F(x,y)}{(D - mD')} = \int F(x, c - mx)\, dx$.

1. Solve $(D^2 - DD' - 2D'^2)$ Z = $(y - 1)e^x$ (Feb2014,Dec2016)
2. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2}$ = y cosx (Dec2016,Jan2023)
3. Solve $(D^3 + D^2D' - DD'^2 - D'^3)$ Z = 3 Sin(x+y) (Oct2018)
4. Solve $(D^2 - 2DD' + D'^2)$ Z = 2xcosy (Dec2017)

Non Homogeneous Linear Equations

1. Solve $(D - D' - 1)(D - D' - 2)$ Z = e^{2x-y} (Dec2016)
2. Solve $(D^2 - DD' - 2D)$ Z = Sin(3x+4y) (Dec2018)
3. Solve $(D^2 - DD' + D' - 1)$ Z = Sin(x+2y) (Dec2017)

4.Solve $(D + D' - 1)(D + 2D' - 3)$ Z = 4+ 3x +6y (Dec2016)

Applications of Partial Differential Equations

Method of separation of variables (2 hrs):

1. Solve the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t}$ +u where u(x,0) = $6e^{-3x}$ by methodofvariationofparameters. (2003,Jan2012,Feb2013,Aug2022,Jan2023)

2. *By the* separationofvariables , find the solution of partial differential equation 2 $\frac{\partial u}{\partial t}$ +3 $\frac{\partial u}{\partial x}$ =3u ,u(x,0)=$4e^{-x}$ (Aug2022,Jan2023)

3. Solve,usingmethodofseparationofvariables,thepartial differential equation $\frac{\partial u}{\partial y}$ +2u = $\frac{\partial^2 u}{\partial x^2}$ givenconditionsareu=0and^{6u}=1+e $-^{3y}$whenx=0forallvaluesofy. (Jan2012,Jan2023)

3. Solve the methodofseparationofvariables $4u_x+u_y$ = 3u and u(0,y) = e^{-5y} (Dec2016,Aug2022,Feb2022)

4. Solve by variable separable method, find all possible solutions of $\frac{\partial^2 z}{\partial u^2} - 2\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$ = 0(Jan2023)

5. Solve the partial differential equation $\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y}$ = 0 and u(0,y) = $8e^{-3y}$ by the methodofvariationofparameters. (Nov2017)

Wave Equation(1 h):

1. Solve the wave equation $\frac{\partial^2 y}{\partial x^2} = c^2\frac{\partial^2 y}{\partial t^2}$ subject to (i) y(0,t) =0 (ii) y(π,t) =0 (iii) y(x,0) =x, o$\le x \le \pi$, $\frac{\partial y}{\partial t}$ (x,0) =0,0$\le x \le \pi$. (2000,May2016,Aug2022,Jan2023)

2. Solve the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$under theconditions y(0,t) =0, y(l,t) = 0 for all t, y(x,0) =f(x) and $(\frac{\partial y}{\partial t})_{t=0}$ = g(x) ,o$< x < l$ (2002,July2022)

3.Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ corresponding to the triangularinitial deflection f(x) = $\begin{cases} \frac{2k}{l}x \; where \; o < x < \frac{l}{2} \\ \frac{2k}{l}(l-x) where \frac{l}{2} < x < l \end{cases}$ (Feb2014,Jan2023)

4. Solve the partial differential equation $\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}$,0$< x < l$ which satisfies the conditions u(0,t) =0,u(l,t) =0 for t$>$ 0.u(x,0) = $\begin{cases} x \; where \; o < x < \frac{l}{2} \\ (l-x) where \frac{l}{2} < x < l \end{cases}$ (Jan2023)

Heat Equation(1 h)

1. An insulated rod of length *l* has its ends A and B maintained at 0^0C and 100^0Crespectively until steady state conditions prevail. If B is suddenly reduced to0^0CandmaintainedatC,0^0findthetemperatureatadistancex fromAattimet. (Feb2022,Jan2023)

2. The ends A and B of a bar 20 cm long have the temperatures 300^oCand 80^oCuntil steadyprevails.Ifthe temperaturesat AandBaresuddenlyreducedto 0^ocandmaintained at0^oC.Find thetemperaturein abar. (Dec2016,Jan2023).

Laplace Equation (1 h)

1. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = 0 with u(0,y) =0, u(x,0) = 0 and u(x,a) = sin($\frac{n\pi x}{l}$) , where 0$\le x \le l$, 0$\le\le y \le a$ and n is positive integer. (June2012,Oct2018)