-Ab- 18, 58

SIR C.R. REDDY COLLEGE OF ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

ACADEMIC YEAR: 2017-18, I-SEMESTER

II-INTERNAL EXAMINATIONS

Subject with code: Fluid Mechanics-I CE 2105 Dt:21 -11-2017.

Class: II/IV B.E-Civil Max. Marks: 30 Time: 2:30 to 4:10 PM

Answer the following

Each question carries 6 marks **5X6=30M**

1. a) A pipe of 1.2m diameter carrying $2.5m^3/s$ of water is deflected through 90° bend. The ends of the bend are anchored by the rods at right angles to the bend (one tie rod at each end) Find the tension in each rod. Also determine the resultant dynamic thrust on the bend & the direction of this thrust.

(OR)

b) Derive the Darcy weisbach equation for friction head loss in a pipe.

2. a) When a sudden contraction is introduced in a horizontal pipe line from 500mm diameter to 250mm diameter, the pressure changes from $105kN/m^2$ to $69kN/m^2$. If the coefficient of contraction is assumed to be 0.65, calculate the water flow rate.

(OR)

b) Derive continuity equation in 3Dimensional Cartesian coordinate system.

3. a) A rectangular tank 6m long, 2m wide & 2m deep contains water upto a depth of 1m. It is accelerated horizontally at $2.5m/s^2$ in direction of its length. Determine **(4M)**
   i) Slope of the free surface
   ii) Maximum & minimum pressure intensities at bottom.

   b) Define stream line & streak line **(2M)**

(OR)

c) Derive Hagen-Poiseullie equation for laminar flow in circular pipes with assumptions.

4. a) A liquid with specific gravity 2.8 & viscosity 0.8 poise flows through a smooth pipe of unknown diameter resulting in a pressure drop of $800N/m^2$ in 2000m length of the pipe. What is the pipe diameter if the mass flow rate is 2500 kg/hour? **(4M)**

   b) Define velocity potential & stream function. **(2M)**

(OR)

c) Prove that the maximum velocity is equal to 1.5 times the average velocity for viscous flow between the two parallel when the both plates are fixed across a section.

5. a) The x & y components of velocity in two dimensional incompressible flow are given by $U = 3x+3y$, $V = 2x-3y$. Derive an expression for the stream function. Show that the flow is rotational & determine the angular velocity in the flow filed. **(4M)**

   b) Define translation and rotation of a fluid element in motion. **(2M)**

(OR)

c) What do you mean by equivalent pipe? Obtain expression for equivalent pipe.

***

IInd Internal EXAM Key

FLUID MECHANICS - I

II/IV B.E — CE2105

1) A pipe of 1.2m diameter carrying 2.5 m³/s of water is deflected through a 90° bend. The ends of the bends are anchored by the rods at right angles to the bend. Find the tension in each rod. Also determine the resultant dynamic thrust on the bend and the direction of this thrust.

Given data

Diameter of the pipe (D) = 1.2m

Discharge through pipe = $2.5\ m^3/s$

Angle of bend = 90°

Area of pipe $A_1 = A_2 = \frac{\pi}{4} \times (1.2)^2 = 1.13 m^2$

$$Q = AV$$

$$V_1 = V_2 = \frac{Q}{A} = \frac{2.5}{1.13} = 2.21 m/s$$

forces on the bend in x direction

$$F_x = \rho Q [V_1 - V_2 \cos\theta] + P_1 A - P_2 A_2 \cos\theta$$

$$= 1000 \times 2.5 [2.21 - 2.21 \cos 90]$$

$$= 5525 N$$

forces on the bend y direction

$$F_y = \rho Q [-V_2 \sin\theta] - P_2 A_2 \sin\theta$$

$$= 1000 \times 2.5 [-2.21 \sin 90°]$$

$= -5525 N$

Resultant dynamic thrust $F_R = \sqrt{F_x^2 + F_y^2}$

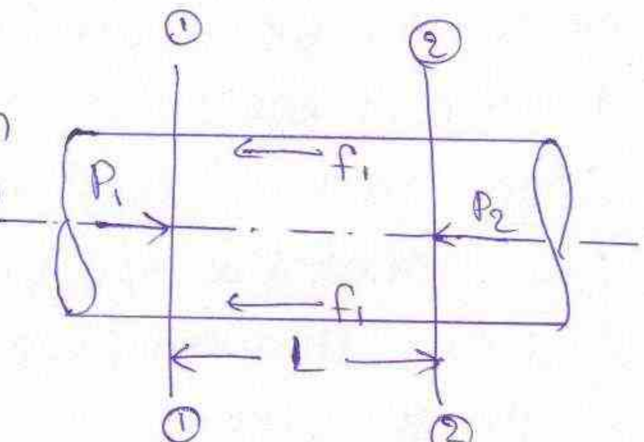
$= 7813.52 N$

Direction of resultant thrust $\tan\theta = \frac{F_y}{F_x}$

$\theta = \tan^{-1}\left[\frac{5525}{-5525}\right]$

$\theta = -45^\circ$

b) Derive Darcy weisbach equation fol friction loss in a pipe



consider a pipe of length L. consider two sections ① & ②. The length of the pipe between the sec ① & ② is L.

lets

P_1 = pressure intensity at sec ①-①

P_2 = pressure intensity @ sec ②-②

V_1 = velocity of flow at sec ①

V_2 = velocity of flow at sec ②

d = diameter of pipe

f_1^* = frictional resistance per unit wetted area per unit velocity

h_f = Loss of head due to friction.

Applying Bernoullis equation between sec① & sec②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since the pipe is horizontal $z_1 = z_2$

Diameter of the pipe is constant hence $V_1 = V_2$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\therefore h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}$$

frictional resistance $F_1 = f' \times \pi d \times L \times V^2$

$$= f' \times P \times L \times V^2$$

forces acting on fluid between sec ① & ② are

i) pressure force at sec ① — $P_1 \times A$

ii) pressure force at sec ② — $P_2 \times A$

iii) frictional force F_1

Resolving all forces in horizontal direction

$$P_1 A - P_2 A - F_1 = 0$$

$$(P_1 - P_2) A = f' \times P \times L \times V^2$$

$$P_1 - P_2 = \frac{f' \times P \times L \times V^2}{A}$$

$$P_1 - P_2 = \rho g h_f$$

equating above equations

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f' \times P \times L \times V^2}{A \times \rho \times g}$$

but $\frac{P}{A} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

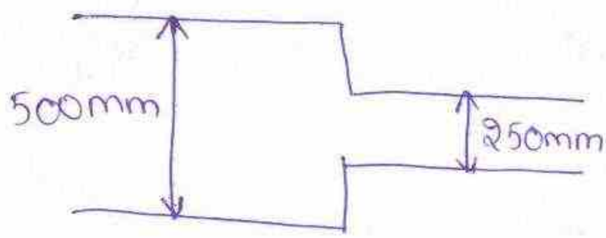
$$\therefore h_f = \frac{4f'LV^2}{\rho g d}$$

but $\frac{f'}{\rho} = \frac{f}{2}$

$$\therefore h_f = \frac{4flv^2}{2gd}$$

II, when a sudden contraction is introduced in a horizontal pipe from 500mm diameter to 250mm diameter the pressure changes from 105 KN/m² to 69 KN/m². If the coefficient of contraction is assumed to be 0.65. calculate the water flow rate.

Given data



Diameter of pipe @ sec ① = 500mm

Area of pipe @ sec ① $= \frac{\pi}{4} \times (0.5)^2 = 0.196\,m^2$

Diameter of pipe @ sec ② = 250mm

Area of pipe @ sec ② $= \frac{\pi}{4} \times (0.25)^2 = 0.049\,m^2$

pressure intensity @ sec ① = P_1 = 105 KN/m²

pressure intensity @ sec ② = P_2 = 69 KN/m²

coefficient of contraction C_c = 0.65

Head loss due to contraction $= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$

$$= \frac{V_2^2}{2\times 9.81}\left[\frac{1}{0.65} - 1\right]^2$$

$$= 0.289\,\frac{V_2^2}{2g}$$

from continuity equation $A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2 V_2}{A_1}$$

$$= \frac{0.049 \times V_2}{0.196}$$

$$V_1 = \frac{V_2}{4}$$

Applying Bernoullis eqn b/t sec① & sec②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c$$

$$\frac{105\times 10^3}{10^3\times 9.81} + \frac{V_2^2}{32\times 9.81} = \frac{69\times 10^3}{10^3\times 9.81} + \frac{V_2^2}{2g} + 0.289\,\frac{V_2^2}{2g}$$

$$105 + \frac{V_2^2}{32} = 69 + \frac{V_2^2}{2} + 0.289\,\frac{V_2^2}{2}$$

$$36 = 0.613\,V_2^2$$

$$V_2 = 7.66\ m/s$$

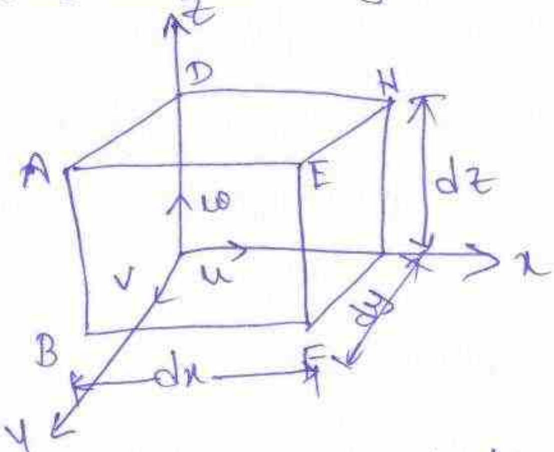
Loss due to contraction $= \frac{(7.66)^2}{2\times 9.81}\left[\frac{1}{0.65} - 1\right]^2$

$$= 0.864\ m$$

Rate of flow of water $Q = A_2 V_2 = 0.049 \times 7.66$

$$= 0.375\ m^3/s$$

2) Derive continuity equation in three dimensional cartesian coordinate system.



consider a fluid element of length dx, dy, dz in x, y & z directions.

let u, v, w are the velocity components in x, y, z directions.

mass of fluid entering the face ABCD / sec

$= \rho \times$ velocity in x direction $\times$ Area

$= \rho \times u \times (dy \times dz)$

mass of fluid leaving the face EFGH / sec

$= \rho u dy dz + \frac{\partial}{\partial x}[\rho u dy dz] dx$

Gain of mass in x direction = mass through ABCD − mass through EFGH per sec

$= \rho u dy dz - \rho u dy dz - \frac{\partial}{\partial x} \rho u dx dy dz$

$= -\frac{\partial}{\partial x} \rho u dx dy dz$

similarly Gain of mass in y direction $= \frac{-\partial}{\partial y} \rho v dx dy dz$

Gain of mass in z direction $= -\frac{\partial}{\partial z} \rho w dx dy dz$

$\therefore$ Net gain of mass $= -\left[\frac{\partial}{\partial x} \rho u + \frac{\partial}{\partial y} \rho v + \frac{\partial}{\partial z} \rho w\right] dx dy dz$

The net gain of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.

But mass of fluid in the element is $\rho\, dx dy dz$ & its rate of increase within the time is

$$\frac{\partial}{\partial t}(\rho\, dx dy dz)$$

Equating above expressions

$$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right] dx dy dz = \frac{\partial \rho}{\partial t} dx dy dz$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

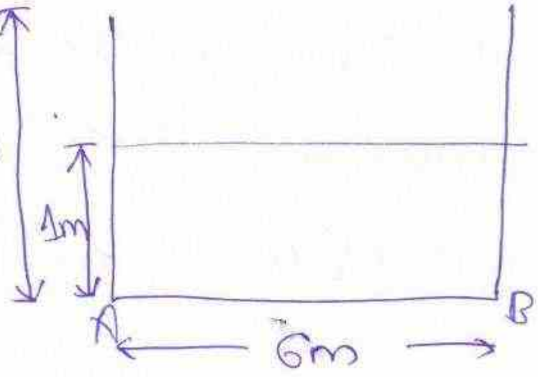
For a steady incompressible flow, continuity equation is given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

③ i) A rectangular tank 6m long, 2m wide & 2m deep contains water upto a depth of 1m. It is accelarated horizontally at 2.5 m/s² in the direction of its length. Determine

i) slope of the free surface

ii) maximum & minimum pressure intensities at bottom.



Given data

Accelaration $a = 2.5 m/s^2$

Length of the tank = 6m

width " " = 2m

depth " " = 2m

i) The angle of the water surface to the horizontal

$$\tan\theta = -\frac{a}{g}$$

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

$$= \tan^{-1}\left(\frac{2.5}{9.81}\right)$$

$$\theta = 14.29^\circ$$

maximum & minimum pressure intensities at the bottom of the tank

Depth of water at front end $= 1 - 3\tan\theta$

$= 1 - 3\tan(14.29) = 0.235m$

Depth of water at rear end $= 1 + 3\tan\theta = 1.764m$

pressure intensity @ the bottom point A is given by

$$P_{max} = \rho g h$$

$$= 1000 \times 9.81 \times 1.764$$

$$= 17304.84 \ N/m^2$$

$$P_{min} = \rho g h$$

$$= 1000 \times 9.81 \times 0.235$$

$$= 2305.35 \ N/m^2$$

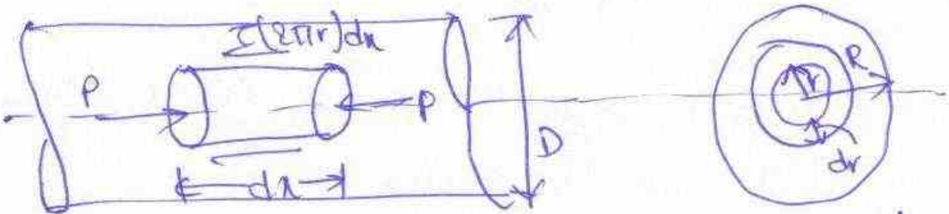
b) stream line

A stream line is an imaginary line within the flow so that the tangent at any point on it gives the velocity of flow at that point

Streak line

It is a curve which gives an instantaneous picture of the location of the fluid particles which have passed through a given point.

2) Derive Hagen-poiseullie equation for laminar flow in circular pipes with assumptions.



consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right. consider a fluid element of radius r. let the length of fluid element is Δx.

forces acting on the fluid element are

i) pressure force $p \times \pi r^2$ on face AB

ii) pressure force $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD

iii) Shear force $\tau \times 2\pi r \Delta x$ on surface of the element

$$p \times \pi r^2 - \left[p + \frac{\partial p}{\partial x}\Delta x\right]\pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial p}{\partial x} \cdot r - 2\tau = 0$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

Velocity distribution

The value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in the above equation.

$$y = R - r$$

$$dy = -dr$$

$$\therefore \tau = \mu\left(\frac{du}{-dr}\right)$$

$$-\mu\frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{r}{2}$$

Integrating above eqn w.r.t r

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}r^2 + C$$

from boundary conditions $r = R$, & $u = 0$

$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 + C$$

$$C = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

substitute 'c' value in the above eqn.

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x}R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right]$$

The above eqn represents eqn of parabola & hence velocity distribution across a section of a pipe is parabolic.

The average velocity $\bar{u}$ is obtained by considering the flow through a circular ring element of radius r & thickness dr. The fluid flowing per sec is given by

$$dQ = u \times 2\pi r\,dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r\,dr$$

$$\therefore Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] 2\pi r\,dr$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2}$$

$$\therefore \bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

Drop of pressure for a given length (L) of pipe

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$$

$$-\left(\frac{\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating the above eqn w.r.t x we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$-[P_1 - P_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2]$$

$$P_1 - P_2 = \frac{8\mu\bar{u}}{R^2}[L]$$

$$P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}$$

$\therefore$ Loss of pressure head $\frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2}$

The above eqn is called as Hagen poiseullie eqn.

4 A) A liquid with specific gravity 2.8 & viscosity 0.8 poise flows through a smooth pipe of unknown diameter, resulting in a pressure drop of 800 N/m² in 2000m length of pipe. what is the pipe diameter if the mass flow rate is 2500 kg/hour.

Given data

Specific gravity of liquid	$= 2.8$
Viscosity of liquid	$= 0.8$ poise $= \frac{0.8}{10} = 0.08 \frac{Ns}{m^2}$
pressure drop	$= 800 N/m^2$
length of the pipe	$= 2000m$
mass flow rate	$= 2500 kg/hour$

from Hagen-poisuellie eqn

$$P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}$$

mass of oil per sec $= \frac{2500}{60 \times 60}$

$= 0.694 \text{ kg/s}$

$$\rho = \frac{m}{V}$$

mass $= \rho \times V$

mass/sec $= \frac{\rho \times V}{t}$

$= \rho \times Q$

$0.694 = 2.8 \times 1000 \times Q$

$\therefore Q = 2.47 \times 10^{-4} \text{ m}^3/\text{s}$

$\therefore$ Avg velocity $\bar{u} = \frac{Q}{A}$

$= \frac{2.47 \times 10^{-4}}{\frac{\pi}{4} \times D^2}$

$$P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}$$

$$800 = \frac{32 \times 0.08 \times 2.47 \times 10^{-4} \times 2000}{\frac{\pi}{4} \times D^4}$$

$D = 0.211 \text{m}$

$\therefore$ Diameter of the pipe $= 0.211 \text{m}$

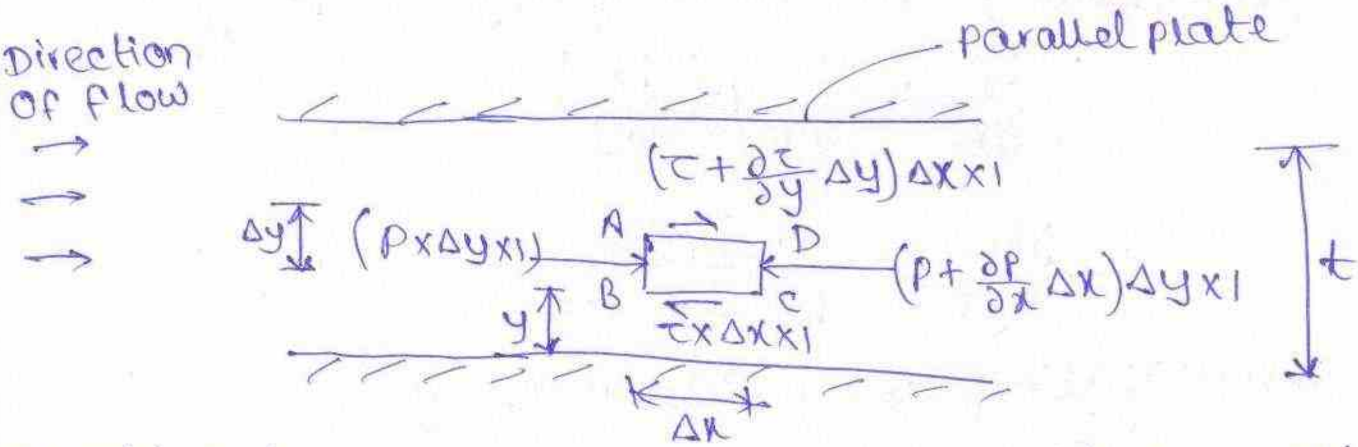
Velocity potential function (ϕ)

It is defined as scalar function of space & time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

Stream function (ψ)

It is defined as scalar function of space & time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction.

2) Prove that the maximum velocity is equal to 1.5 times the average velocity for viscous flow between the two parallel plates when the both plates are fixed across a section.



consider two parallel plates fixed at a distance 't' apart. consider a viscous fluid is flowing in between them from left to right. consider a fluid element of length Δx & thickness Δy at a distance 'y' from the lower fixed plate.

The forces acting on the fluid element are

1. pressure force $p \times \Delta y \times 1$ on face AB
2. pressure force $(p + \frac{\partial p}{\partial x}\Delta x)\Delta y \times 1$ on face CD
3. shear force $\tau \times \Delta x \times 1$ on face BC
4. shear force $(\tau + \frac{\partial \tau}{\partial y}\Delta y) \times \Delta x \times 1$ on face AD

for steady uniform flow, there is no accelaration & hence the resultant force is zero.

$$p \times \Delta y \times 1 - (p + \frac{\partial p}{\partial x}\Delta x)\Delta y \times 1 - \tau \times \Delta x \times 1 + (\tau + \frac{\partial \tau}{\partial y}\Delta y)\Delta x = 0$$

$$-\frac{\partial p}{\partial x}\Delta x \Delta y + \frac{\partial \tau}{\partial x}\Delta y \times \Delta x = 0$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

Velocity distribution

from newton's law of viscosity $\tau = \mu\left(\frac{du}{dy}\right)$

$$\therefore \frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\mu\left(\frac{du}{dy}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating above eqn w.r.t y

$$\int \frac{\partial u}{\partial y} = \int \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1$$

$$u = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

from boundary conditions i) @ $y=0, u=0$

ii). @ $y=t, u=0$

substituting in above eqn

$C_2 = 0$

$C_1 = \frac{-1}{2\mu}\frac{\partial p}{\partial x} t$

$\therefore u = \frac{1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2]$

Ratio of max to avg velocity

Velocity is maximum @ $y = t/2$

$\therefore u_{max} = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[t\frac{t}{2} - \frac{t^2}{4}\right]$

$u_{max} = \frac{-1}{8\mu}\frac{\partial p}{\partial x} t^2$

Avg velocity ($\bar{u}$) is obtained by dividing the discharge (Q) across the section by the area of the section ($t \times 1$) and the discharge Q is obtained by considering the rate of flow of fluid through the strip of thickness dy & indegrating it

$$dQ = \frac{-1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$Q = \int_0^t dQ = \int_0^t \frac{-1}{2\mu}\frac{\partial p}{\partial x}[ty - y^2] \times dy \times 1$$

$$= \frac{-1}{12\mu}\frac{\partial p}{\partial x} t^3$$

$$\bar{u} = \frac{Q}{A} = \frac{-\frac{1}{12\mu}\frac{\partial p}{\partial x} t^3}{t \times 1} = \frac{-1}{12\mu}\frac{\partial p}{\partial x} t^2$$

$$\therefore \frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x} t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x} t^2} = \frac{3}{2} = 1.5$$

5) The velocity components in a two dimensional flow field for an incompressible fluid are expressed as

$u = \frac{3x}{} + 3y -$ $\qquad$ $V = 2x - 3y$

i) show that these functions represents a possible case of irrotational flow.

ii) obtain the expression for stream function ψ

iii) obtain the expression for velocity potential ϕ

Given $u = 3x + 3y$ $\quad$ $V = 2x - 3y$

from definition $\frac{\partial \psi}{\partial x} = V$ $\quad$ $\frac{\partial \psi}{\partial y} = -u$

$$2x - 3y = \frac{\partial \psi}{\partial x}$$

$$(2x - 3y)\partial x = \partial \psi$$

Integrating above eqn w.r.t x

$$x^2 - 3xy + k = \psi$$

Differentiating above eqn w.r.t y

$$\frac{\partial \psi}{\partial y} = -3x + \frac{\partial k}{\partial y}$$

$$-3x - 3y = -3x + \frac{\partial k}{\partial y}$$

$$(-3y)\partial y = \partial k$$

Integrating above eqn w.r.t y

$$-3 \cdot \frac{y^2}{2} = k$$

sub k value in above eqn

$$\therefore \psi = x^2 - 3xy - \frac{3}{2}y^2$$

for irrotational flow, it has to satisfy laplace eqn ie $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

$$\therefore \frac{\partial^2}{\partial x^2}\left(x^2 - 3xy - y^2\tfrac{3}{2}\right) + \frac{\partial^2}{\partial y^2}\left(x^2 - 3xy - \tfrac{3}{2}y^2\right) = 0$$

$$2 - 3 = 0$$

$$-1 \neq 0$$

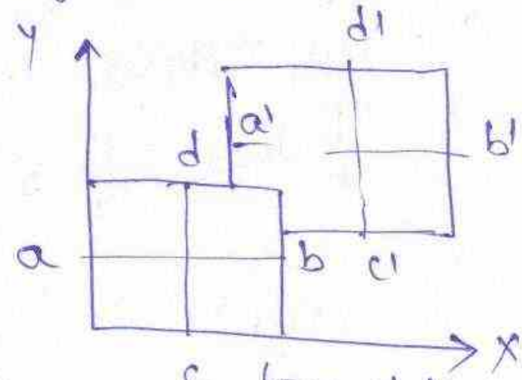
Hence flow is irrotational

Angular velocity $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

$$\omega_z = -0.5 \text{ units/sec}$$

b) Translation

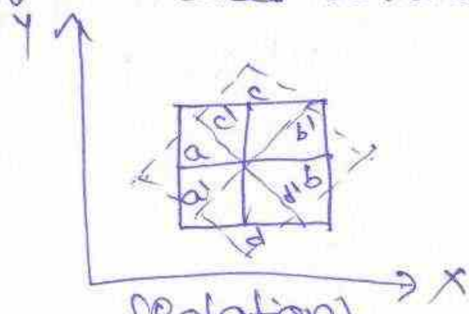
It is defined as the movement of a fluid element in such a way that it moves bodily from one Poisition to another position & the two axes ab & cd represented in new position by a'b' & c'd' are parallel



(Translation)

Rotation

It is defined as the movement of fluid element in such a way that both of its axes rotate in same direction.



(Rotation)

1 what do you mean by Equivalent pipe? obtain Expression for equivalent pipe.

Equivalent pipe:-

It is defined as the pipe of uniform diameter having loss of head & discharge equal to the loss of head & discharge of a compound pipe consisting of several pipes of different lengths & diameters.

The length of equivalent pipe is equal to the sum of lengths of the compound pipe consisting on different pipes.

$$L = L_1 + L_2 + L_3$$

Total head loss in compound pipe after neglecting minor losses.

$$H = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

Assuming $f_1 = f_2 = f_3 = f$

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$Q = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2 = \frac{\pi}{4}d_3^2V_3$$

$$\therefore V_1 = \frac{4Q}{\pi d_1^2}, \quad V_2 = \frac{4Q}{\pi d_2^2}, \quad V_3 = \frac{4Q}{\pi d_3^2}$$

Sub these values in above eqn we get

$$\therefore H = \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \quad \text{—} (1)$$

Head loss in equivalent pipe $H = \frac{4fLV^2}{2gd}$

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$\therefore H = \frac{4 \times 16 \times Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right] \quad \text{---} \quad (2)$$

equating eq(1) & (2)

$$\frac{4 \times 16 \times Q^2 \times f}{\pi^2 \times 2g}\left[\frac{L}{d_1^5} + \frac{L}{d_2^5} + \frac{L}{d_3^5}\right] = \frac{4 \times 16 \times Q^2 \times f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

$$\boxed{\frac{L}{d_1^5} + \frac{L}{d_2^5} + \frac{L}{d_3^5} = \frac{L}{d^5}}$$